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TURBULENT FLOW IN A BOUNDARY LAYER ON THE INLET  
AND OUTLET SIDES OF A ROTATING CHANNEL

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UDC 532.526.4:621.515

Expressions are obtained for approximately determining the shear stresses on the outlet and inlet sides of a rotating channel on the basis of a turbulence energy balance equation, A. N. Kolmogorov's hypothesis, and the Monin-Obukhov similitude theory.

In the flow of a fluid in the rotating channels of turbine rotors, body forces are created by the rotation and curvature of the channel walls. As an example, Fig. 1 shows body forces acting on a particle of fluid on the pressure side of a blade (the outlet side of the channel) in a plane impeller in a radial-flow compressor. The x axis is directed along the blade surface, the y axis is normal to the surface, and the z axis is parallel to the angular velocity vector. It can be seen that in most cases the total body force is negative on the pressure side and positive on the suction side of the blade (the inlet side of the channel).

The different directions of the total body forces on the pressure and suction sides determines the different character of flow in the turbulent boundary layer.

We can use Rayleigh's method to evaluate the stability of the flow and take the Richardson number as the criterion of stability [1, 2]. The total body force acting in the direction of the y axis is equal to (Fig. 1)

$$F = \mp \rho \left( 2 \omega u \pm \frac{u^2}{R} - \omega^2 r \cos \eta \right), \quad (1)$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 6, pp. 977-986, December, 1981. Original article submitted November 10, 1980.

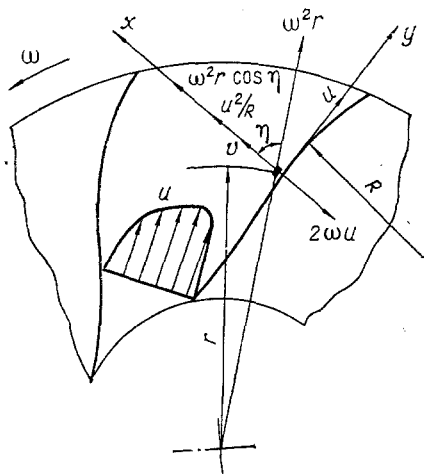


Fig. 1

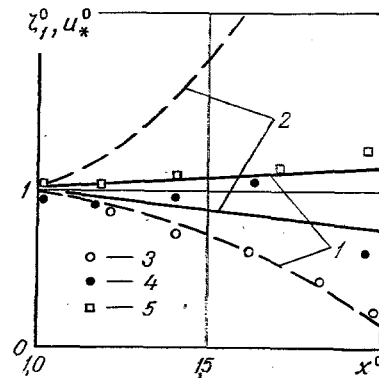


Fig. 2

Fig. 1. Body forces acting on a particle in a boundary layer on the outlet side of a channel between impeller blades.

Fig. 2. Distribution of dimensionless Monin-Obukhov scale and dimensionless dynamic velocity along a rotating channel. Theoretical data of the author for a square channel with 58-mm sides,  $\omega = 40$  1/sec: 1) change in  $u_*^0$ ; 2) change in  $\zeta_0^1$ ; solid lines — for the outlet side, dashed lines — for the inlet side. Experimental data for  $u_*^0$ : 3) inlet side, data from [10]; 4, 5) inlet and outlet sides, respectively, author's data.

where the minus sign in front of the parentheses pertains to the outlet side of the channel and the plus sign denotes the inlet side of channel. The signs in front of  $u^2/R$  are determined by the direction of curvature of the wall. In the simplest case of flow in a radial channel,  $F = \mp 2\rho\omega u$ .

Analyzing the motion of particles in the boundary layer in the field of the acting total body force (variable over the thickness of the boundary layer due to the change in  $u$ ), with random displacements in the direction of the  $y$  axis, it can be shown that if the total body force is negative the flow will be unstable. Conversely, with a positive direction of the body force, the flow in the boundary layer will be stabilized. Let us determine the Richardson number, having represented it as the ratio of the work of the buoyancy forces (expulsive forces) to the work of the frictional forces [1, 3].

With random displacement of a particle by the amount  $\Delta y = l'$ , the buoyancy force acting in the direction of the  $y$  axis will be

$$\Delta F' = \frac{\partial F}{\partial y} \Delta y = \frac{\partial F}{\partial y} l'. \quad (2)$$

The second work of this force

$$A_1' = v' \Delta F' = v' l' \frac{\partial F}{\partial y}, \quad (3)$$

where  $v'$  is the transverse velocity pulsation.

Taking the average, we obtain the work of the buoyancy forces with random displacements

$$A_1 = \langle l' v' \rangle \frac{\partial F}{\partial y}, \quad (4)$$

where the correlation  $\langle l' v' \rangle$  is the coefficient of turbulent transport [1], which we will assume to be equal to  $K$ . Then

$$A_1 = K \frac{\partial F}{\partial y}. \quad (5)$$

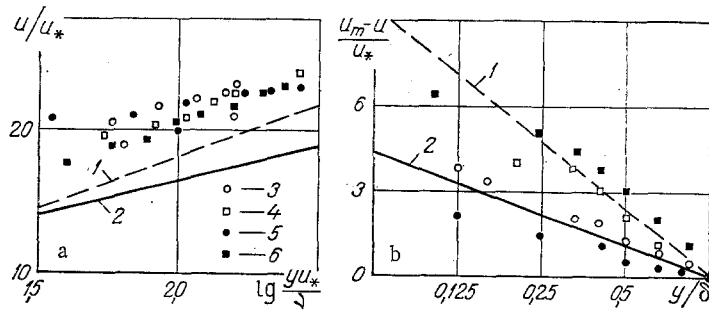


Fig. 3. "Wall law" profiles (a) and "velocity defect" profiles (b) for a rotating square channel with 58-mm sides,  $\omega = 40$  1/sec. Theoretical data: 1) inlet side; 2) outlet side. Experimental data: 3, 4) outlet and inlet sides, respectively, for the initial section; 5, 6) outlet and inlet sides, respectively, for the final section.

The more rigorous result (4) may be obtained by following E. Richardson's arguments [3]. The work of frictional forces in the turbulent boundary layer may be written as follows

$$A_2 = \tau \frac{\partial u}{\partial y} = \rho K \left( \frac{\partial u}{\partial y} \right)^2. \quad (6)$$

We will determine the Richardson number

$$Ri = \frac{A_1}{A_2} = \frac{\partial F / \partial y}{(\partial u / \partial y)^2}. \quad (7)$$

In a gravitational field  $\frac{dF}{dy} = -g \frac{d\rho}{dy}$ , and we obtain the familiar formula for Ri in a stratified medium.

In several cases the Richardson number is expressed through the Brunt-Weissel frequency (buoyancy frequency) [4]:

$$Ri = \frac{\omega_{BV}^2}{(\partial u / \partial y)^2}, \quad (8)$$

which may be determined as

$$\omega_{BV}^2 = \frac{1}{\rho} \frac{\partial F}{\partial y}. \quad (9)$$

Differentiating (1) and using (7), we may obtain the Richardson number for flow in a rotating channel of arbitrary form. For the case of flow of an incompressible fluid in a radial channel, we obtain

$$Ri = \mp \frac{2\omega}{\partial u / \partial y}. \quad (10)$$

In order to henceforth avoid differences in sign, we will adopt a positive value of  $\omega$  for the outlet side of the channel and a negative value for the inlet side. We then write the Richardson number in the form

$$Ri = - \frac{2\omega}{\partial u / \partial y}. \quad (11)$$

It can be seen from (11) that, in the case of a radial channel being discussed, the Richardson number is negative for the outlet side of the channel — where turbulence is intensified — and positive for the inlet side of the channel — where the flow is stabilized.

To further analyze flow, we will write the turbulence energy balance equation in the form [1]

$$\frac{\partial}{\partial x_\alpha} \left[ \frac{1}{2} \langle \rho u'_\alpha u'_\beta u'_\beta \rangle + \langle p' u'_\alpha \rangle \right] + \frac{\partial}{\partial x_\alpha} [\rho b u_\alpha] = \langle \rho u'_\alpha \rangle \left( X_\alpha - \frac{Du_\alpha}{Dt} \right) - \langle \rho u'_\alpha u'_\beta \rangle \frac{\partial u_\beta}{\partial x_\alpha} - \rho \epsilon_t. \quad (12)$$

We will successively determine all of the terms of Eq. (12), expressing them through the coefficient of turbulent viscosity  $K$  in accordance with the hypothesis of A. N. Kolmogorov.

Let us examine the first term of the equation, which determines the transport of turbulent energy by the velocity pulsation and the work of the pressure forces:

$$\frac{\partial}{\partial x_\alpha} \left[ \frac{1}{2} \langle \rho u'_\alpha u'_\beta u'_\beta \rangle + \langle p' u'_\alpha \rangle \right] = - \frac{\partial}{\partial x_\alpha} \left( \alpha_b K \frac{\partial b}{\partial x_\alpha} \right). \quad (13)$$

We will use the relations given in [1] for  $b$  and  $l$ :

$$b = \frac{K^2}{l^2}, \quad l = \frac{\kappa \lambda y}{c_0}. \quad (14)$$

Differentiating  $b$  with respect to  $x$  and  $y$  and assuming  $\lambda \sim \text{const}$  in a first approximation, we obtain

$$\begin{aligned} \frac{\partial b}{\partial y} &= 2 \left( \frac{c_0}{\kappa \lambda y} \right)^2 K^2 \left[ \frac{\partial K}{\partial y} - \frac{K}{y} \right], \\ \frac{\partial b}{\partial x} &= 2 \left( \frac{c_0}{\kappa \lambda y} \right)^2 K \frac{\partial K}{\partial x}. \end{aligned}$$

Let us examine the second term of Eq. (12), determining the transport of turbulent energy by the mean velocity. Here, as usual, it is assumed that the transverse mean velocity is trivial:

$$\frac{\partial}{\partial x_\alpha} (\rho u_\alpha b) = \rho K \left( \frac{c_0}{\kappa \lambda y} \right)^2 \left[ K \frac{\partial u}{\partial x} + 2u \frac{\partial K}{\partial x} \right]. \quad (16)$$

The third term of Eq. (12) gives the work of the buoyancy forces with turbulent displacements in a compressible fluid [1]. We could use Eq. (5) to determine it, without making additional assumptions regarding the relationship between the correlations  $\langle \rho u' \rangle$ ,  $\langle \rho v' \rangle$  and the mean parameters of the flow. However, besides body forces, the third term of Eq. (12) also contains the derivatives  $Du_\alpha/Dt$ . We will write the expressions for  $\langle \rho u' \rangle$  and  $\langle \rho v' \rangle$  in the form [1, 5]

$$\langle \rho u' \rangle = \frac{\rho}{\gamma p} \langle p' u' \rangle, \quad \langle \rho v' \rangle = \frac{\rho}{\gamma p} \langle p' v' \rangle, \quad (17)$$

here assuming that

$$\langle p' u' \rangle = \rho \sigma_1 b u, \quad \langle p' v' \rangle = \rho \sigma_2 b u, \quad (18)$$

where  $\sigma = 0.1-0.01$ . We could also take an expression of type (13) for  $\langle p' u' \rangle$  and show that we would arrive at the same final results as with the use of (18).

From the equation of motion for a plane rotating channel, we may obtain

$$X - \frac{Du}{Dt} = \omega^2 x - u \frac{\partial u}{\partial x}, \quad Y - \frac{Dv}{Dt} = -2 \omega u. \quad (19)$$

Assuming  $\sigma_1 \sim \sigma_2$  in a first approximation and allowing for (18) and (19), we obtain the third term of Eq. (12) in the form

$$\langle \rho u_\alpha \rangle \left( X_\alpha - \frac{Du_\alpha}{Dt} \right) = \frac{\rho \sigma u K^2}{a^2} \left( \omega^2 x - u \frac{\partial u}{\partial x} - 2 \omega u \right) \left( \frac{c_0}{\kappa \lambda y} \right)^2. \quad (20)$$

Supposing that

$$\langle u' v' \rangle = -K \frac{\partial u}{\partial y}, \quad (21)$$

and ignoring the quantity  $dv/dy$ , we have the following expression for the fourth term of Eq. (12):

$$\langle \rho u'_\beta u'_\beta \rangle \frac{\partial u_\beta}{\partial x_\alpha} = \rho K \left( \frac{\partial u}{\partial y} \right)^2. \quad (22)$$

We will write the turbulence energy dissipation in the form [1]

$$\rho \varepsilon_t = \frac{\rho b^{3/2}}{c_0^4 l} = \frac{\rho K^3}{(\kappa \lambda y)^4}. \quad (23)$$

Using the corrected relations, from balance equation (12) we obtain the following differential equation for the coefficient of turbulent viscosity  $K$ :

$$y^2 \frac{\partial}{\partial y} \left[ \frac{K^2}{y^2} \left( \frac{\partial K}{\partial y} - \frac{K}{y} \right) \right] - \frac{1}{K} \frac{\partial}{\partial x} \left[ K^2 \left( 2 \alpha_b \frac{\partial K}{\partial x} - u \right) \right] = K \sigma M^2 \Phi + \left( \frac{\partial u}{\partial y} \right)^2 - \left( \frac{K}{\kappa \lambda y c_0} \right)^2, \quad (24)$$

where  $\Phi = \omega^2 x/u - \partial u/\partial x - 2\omega$ .

Considering that the mechanism by which the body forces act on the flow in the boundary layer of a rotating channel is analogous to the effect of stratification in a gravitational field, to further transform Eq. (24) we will use the results of the similitude theory of Monin—Obukhov for a stratified medium [1]. This makes it possible to describe all of the important characteristics of the turbulent boundary layer by means of universal functions dependent on the dimensionless parameter  $\zeta$ :

$$\zeta = y/L, \quad (25)$$

where

$$L = - \frac{\rho u_*^3}{\kappa B}; \quad (26)$$

$B$  is the work of buoyancy forces in the stratified medium.

Using (1) and (5), we obtain the work of buoyancy forces for the case of flow of an incompressible fluid in a rotating channel

$$A_1 = K \frac{\partial F}{\partial y} = 2 \rho K \left( \omega \pm \frac{u}{R} \right) \frac{\partial u}{\partial y}. \quad (27)$$

Representing the coefficient of turbulent viscosity in the form

$$K = \frac{u_*^2}{\partial u/\partial y}, \quad (28)$$

and using (27), we can — on the basis of Eq. (26) — obtain the following expression for the Monin—Obukhov length scale in the case of flow in a rotating channel:

$$L = - \frac{u_*}{\kappa \Omega} \quad (29)$$

and obtain the following expression for the dimensionless scale

$$\zeta = - \frac{\kappa \Omega y}{u_*}, \quad (30)$$

where  $\Omega = \omega \pm u/R$ . The coefficient 2 has been omitted for convenience.

Scale  $L$  determined by Eq. (29) is in essence equal to scale  $H$  obtained in [1] for the planetary boundary layer, turbulent flow in which is determined by the dynamic velocity and the Coriolis parameter.

Let us transform Eq. (24), expressing all of its terms through the dimensionless scale  $\zeta$ . Following [1], we will take the following as the expression for the velocity profile

$$\frac{\partial u}{\partial y} = \frac{u_*}{\kappa y} \varphi(\zeta) \quad (31)$$

and the following linear approximation for  $\varphi(\zeta)$

$$\varphi(\zeta) = 1 + \beta \zeta, \quad (32)$$

valid at small  $\zeta$ , where  $\beta$  is an empirical coefficient. Having taken  $\Omega = \omega$  ( $u/R \sim 0$ ), from (30) and (31) we obtain

$$\frac{\partial u}{\partial y} = \frac{u_*}{\kappa y} (1 + \beta \zeta) = - \frac{\omega}{\zeta} (1 + \beta \zeta). \quad (33)$$

Allowing for (33), from (28) we obtain

$$K = -\frac{u_*^2 \zeta}{\omega(1 + \beta\zeta)}. \quad (34)$$

Differentiating (34) with respect to  $x$  while assuming that  $u_* = u_*(x)$ , we write

$$\frac{1}{K} \frac{\partial K}{\partial x} = -\frac{1 + 2\beta\zeta}{\zeta(1 + \beta\zeta)} \frac{\partial \zeta}{\partial x}. \quad (35)$$

We determine the velocity  $u$  by integrating (33)

$$\frac{u}{u_*} = \frac{1}{\kappa} \left[ \ln \frac{\zeta}{\zeta_a} - \beta(\zeta - \zeta_a) \right], \quad (36)$$

where  $\zeta_a$  corresponds to  $y_a$  when  $u = 0$ . As  $y_a$ , we take [6]:

$$y_a = \frac{\alpha v}{u_*}, \quad (37)$$

where

$$\alpha = \exp 5.5 \kappa \sim 1/9.$$

To possibly determine  $u_*$ , let us henceforth examine the flow on the boundary of a viscous sublayer, when  $y = y_1$  ( $\zeta = \zeta_1$ ). We will determine the thickness of the viscous sublayer from the "joining" condition, as given, for example, by Ginsburg [7]. We obtain

$$y_1 = \frac{\alpha_1 v}{\kappa u_*} (1 + \beta\zeta_1) \quad (38)$$

or approximately at  $\alpha_1 = 4.8$

$$u_*^2 = -\frac{12 \kappa \omega v}{\zeta_1}. \quad (39)$$

We will determine the function  $\lambda$  using the experimental data in [8], where  $l/l_0 = 1 - \beta Ri$  for the outlet side of the channel and  $l/l_0 = (1 + \beta Ri)^{-1}$  for the inlet side. From (11), (31), and (32), we obtain the Reynolds number in the form

$$Ri = \frac{\zeta}{\varphi(\zeta)} = \frac{\zeta}{1 + \beta\zeta}. \quad (40)$$

We then have  $l/l_0 = 1 - \beta\zeta$  for the outlet side of the channel and  $l/l_0 = (1 + \beta\zeta)^{-1} \sim 1 - \beta\zeta$  for the inlet side.

Taking  $l_0 = \kappa y$  for a stationary channel, from (14) we write

$$\lambda = c_0 \frac{l}{l_0} = c_0 (1 - \beta\zeta). \quad (41)$$

Using the expressions obtained for  $u/u_*$  and  $K$  and their derivatives, Eq. (41), and expansions valid for small  $\zeta$ :

$$1 + \beta\zeta \sim 1, \quad (1 + \beta\zeta)^2 \sim 1 + 2\beta\zeta \sim 1, \quad (1 \pm \beta\zeta)^4 \sim 1 \pm 4\beta\zeta,$$

from (28) we obtain a differential equation for the scale  $\zeta_1$ , taken on the boundary of the viscous sublayer:

$$96 \alpha_b \kappa v \left( \frac{1}{\zeta_1} \frac{d\zeta_1}{dx} \right)^2 + \frac{2u_1}{\zeta_1} \frac{d\zeta_1}{dx} + 16 \beta^2 \omega \zeta_1 - 2 \beta \alpha_b \kappa^2 \omega + \sigma M_1^2 \Phi = 0, \quad (42)$$

where  $M_1 = u_1/a$ ,  $u_1$  is the velocity at the boundary of the viscous sublayer. We determine the velocity  $u_1$  from (36)-(38)

$$\frac{u_1}{u_*} \sim \frac{1}{\kappa} \ln \frac{\zeta_1}{\zeta_a} \sim 12, \quad \frac{du_1}{dx} \sim 12 \frac{du_*}{dx}. \quad (43)$$

Differentiating (39) with respect to  $x$ , we obtain

$$\frac{du_*}{dx} = -\frac{1}{2} \sqrt{\frac{12 \kappa \omega v}{\zeta_1}} \frac{1}{\zeta_1} \frac{d\zeta_1}{dx}. \quad (44)$$

Evaluation of the first term of Eq. (42) shows that it is an order less than the other terms, even at large  $d\zeta_1/dx$ . Considering the smallness of  $M$  at the boundary of the viscous

sublayer, Eq. (42) may be simplified considerably and, with allowance for (43) and (44), reduced to an equation with divisible variables. Allowing for the signs of  $\zeta$  and  $\omega$ , integration of this equation at  $\alpha_b = 1$  yields two equations:

for the outlet side

$$\frac{1}{\sqrt{\zeta_1^0}} - \frac{\sqrt{8\beta\zeta_{01}}}{2\kappa} \ln \left( \frac{\kappa + \sqrt{8\beta\zeta_1}}{\kappa - \sqrt{8\beta\zeta_1}} \frac{\kappa - \sqrt{8\beta\zeta_{01}}}{\kappa + \sqrt{8\beta\zeta_{01}}} \right) = 1 + \frac{\kappa^2\beta\omega\Delta x}{30 u_{*0}}, \quad (45)$$

for the inlet side

$$\frac{1}{\sqrt{\zeta_1^0}} + \frac{\sqrt{8\beta\zeta_{01}}}{\kappa} \left( \operatorname{arctg} \frac{\sqrt{8\beta\zeta_1}}{\kappa} - \operatorname{arctg} \frac{\sqrt{8\beta\zeta_{01}}}{\kappa} \right) = 1 - \frac{\kappa^2\beta\omega\Delta x}{30 u_{*0}}. \quad (46)$$

A single formula gives a rather rough approximation of (45) and (46)

$$\frac{1}{\sqrt{\zeta_1^0}} = u_*^0 = 1 \pm \frac{\kappa^2\beta\omega\Delta x}{30 u_{*0}}, \quad (47)$$

where  $\zeta_1^0 = \zeta_1/\zeta_{01}$ ;  $u_*^0 = u_*/u_{*0}$ ;  $\zeta_{01}$ ,  $u_{*0}$  pertain to the initial section.

Having expressed  $w$  in the form of  $w = m u_{*0}$ , where  $m$  is a coefficient, we write (47) thus

$$u_*^0 = 1 \pm G \operatorname{Ro} \frac{h}{d_h}, \quad (48)$$

where  $\operatorname{Ro} = (\omega d_h)/w$ ,  $h = \Delta x$ ,  $G = (m\kappa^2\beta)/30$ . Essentially, the same formula was obtained in [8] from analyzing test data on losses in rotating channels. To ensure full agreement, we should take  $G(h/d_h) = 1.75$ , so that we obtain  $\beta = 4-5$  at  $h/d_h = 6-8$ . This coincides fully with the experimental data.

Figure 2 shows theoretical values of the scales  $\zeta_1^0$  and shear stresses on the outlet and inlet sides of a square rotating channel with 58-mm sides ( $\square 58$ ) and a length  $h = 570$  mm determined from Eqs. (45), (46) at  $\omega = 40$  1/sec. The figure also shows experimental data obtained by the author (shear stresses measured by Preston's method [9]) and data from [10] obtained for a rotating diffuser. The theoretical value of  $\beta = 4$ .

Figure 3a gives theoretical and measured "wall law" profiles for a  $\square 58$  channel. The velocity in the boundary layer was measured with three-channel probes patterned after the recommendations in [11]. The experimental unit is described in [12]. Figure 3b shows theoretical and measured "velocity defect" profiles.

The divergence of the theoretical velocity profiles from the measured profiles can evidently be attributed to the fact that the theoretical data pertains to flow over a plate rather than flow in a channel, where there are secondary flows. In particular, a flow develops from the inlet to the outlet side of the channel, leading to distortion of the velocity profile.

Equations (46) and (47) show (see Fig. 2) that  $u_*$  may drop to zero on the inlet side of the channel. This may lead to separation of the flow, as confirmed by numerous experiments — especially [10].

#### NOTATION

$K$ , coefficient of turbulent viscosity;  $L$ , length scale of Monin—Obukhov;  $M$ , Mach number;  $\operatorname{Ro}$ , Rossby number;  $\operatorname{Ri}$ , Richardson number;  $R$ , radius of curvature of wall;  $X$ ,  $Y$ , forces acting in the flow;  $\alpha$ , speed of sound;  $b$ , turbulence intensity;  $c_0$ , coefficient;  $d_h$ , hydraulic diameter of channel;  $l$ , mixing length;  $r$ , current rotation radius;  $p$ , pressure;  $t$ , time;  $u$ ,  $v$ , velocity components along  $x$ ,  $y$  axes;  $w$ , mean velocity of flow;  $\alpha$ ,  $\beta$ , coefficients;  $\gamma$ , ratio of heat capacities;  $\delta$ , thickness of boundary layer;  $\varepsilon$ , turbulence energy dissipation;  $\zeta$ , dimensionless scale of Monin—Obukhov;  $\eta$ , angle between velocity vector of particle and circumferential velocity vector;  $\kappa$ , Prandtl—Karman constant;  $\lambda$ ,  $\varphi$ , functions of  $\zeta$ ;  $\nu$ , coefficient of kinematic viscosity;  $\rho$ , density;  $\sigma$ , level of turbulence;  $\omega$ , angular velocity;  $\omega_{\text{By}}$ , Brunt—Weisel frequency.

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### INFLUENCE OF RADIATION ON THE DEGENERATION OF ISOTROPIC TURBULENCE IN HIGH-TEMPERATURE MEDIA

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UDC 532.517.45

It is shown that radiation exerts a distinct influence on the degeneration of isotropic turbulence depending on the vortex size in conformity with the formula obtained for radiant thermal diffusivity.

Sufficiently many journal articles and monographs are devoted to the investigation of radiation interaction with a substance and to questions of the dynamics of a radiating gas. However, the interaction between radiation and turbulence during motion of high-temperature media has been inadequately studied. Meanwhile, as has been shown in [1, 2], this interaction is substantial for a number of problems of practical importance. The influence of radiation on the structure of degenerating isotropic turbulence in compressible high-temperature gases is examined below.

It can be shown [2, 3] that at temperatures to many thousands of degrees the magnitude of the total volume density of radiation for not very rarefied media is small compared to the volume energy density of the particle thermal motion in the medium. This also refers to the so-called radiant pressure which is small compared to the pressure caused by particle motion in a medium under the conditions mentioned. At the same time, because of the high velocity of radiation propagation the radiation energy transfer can be substantially greater than the energy transfer during motion of the medium or motion of the particles in the medium. We shall later limit ourselves to the case when the equation of state of an ideal gas is approximately valid, and the specific heats  $c_p$  and  $c_v$  are separately constant, i.e., are independent of the temperature.

Under the constraints mentioned, the continuity, motion, and energy equations have the form [2, 4]

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